SOME CONDITIONS FOR THE EXISTENCE OF A "PINCH" STRUCTURE OF THE CURRENT SHEET IN PLASMA

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In pulsed plasma accelerators of various configurations it is frequently possible to observe an instability of the current sheet characterized by separation into individual narrow channels (local "pinches"). In this paper the flow of plasma over these channels is qualitatively investigated. It is shown that a large part of the discharge current can flow through a series of narrow channels only when they completely decelerate the plasma flow. This places upper and lower bounds with respect to pressure on the region of existence of the pinch structure, if it is assumed that deceleration by the magnetic field predominates over viscous deceleration. The upper bound is obtained from the condition of deceleration of the plasma by the magnetic field near the pinches, the lower bound from the condition that the ratio of specific heats of the gas γ must be small. As the initial gas pressure decreases, so does the plasma deceleration time, but the characteristic times of the excitation and ionization processes increase; therefore the condition $\gamma \approx 1$ ceases to be satisfied. The results are compared with experiment. The induced currents in the plasma near the pinches are also a reason for the stability of the network of current filaments.

According to [1] a pulsed high-current discharge in a low-pressure gas develops as follows. After breakdown and the attainment of a certain uniform value of the gas conductivity the current continues to increase only in a surface layer (skin effect), which, if the particle density is sufficient, acts as an impermeable piston on the gas in front of it ("snowplow" model). Clearly, to judge from the collapse time of the current cylinder [2], this gives a good picture of the situation in the Z pinch. However, in coaxial and rail guns the gas is by no means always completely raked up by the current sheet (see, for example, [3]). This is also indicated by the observed broad spectrum of plasmoid velocities.

It has been noted in high-speed photographs of pulsed discharges of very different configurations (Z and θ pinches, coaxial and rail plasma guns) that the current sheet is often divided into a series of channels [4–12], through which the main current flows. This follows from their high acceleration as compared with the rest of the plasma [4, 5], from the arrangement of the cathode spots along the electrodes, and from the large value of the plasma density in the channels [11]. This effect is well reproduced from discharge to discharge.

Experiments with the Z pinch [7] have shown that the division of the current sheet into pinches has almost no effect on its velocity, which is in good agreement with calculations based on the "snowplow" model, although the diameter of the channels (determined, it is true, from the luminescence distribution and not the current density) is much less than the distance between them. On the other hand, in rail guns the pinches only slightly entrain the gas filling the tube [11]. The surface layer pinch instability is observed in the pressure range from several mm to several μ Hg [9]. Both pressure limits depend heavily on the nature of the gas in which the discharge takes place, the upper limit being approximately inversely proportional to the molecular weight of the gas [9]. In a rail gun at low initial gas pressures ($p < 20\ \mu$ Hg) pinches appear at some optimal rate of gas release from the electrodes and the walls [12].

A uniform transverse magnetic field has only a slight influence on the pinches, since it is smaller in magnitude than the magnetic field of the pinches themselves [11]. A transverse magnetic field nonuniform along the length of the pinches causes them to decay, if the magnitude of the external field is comparable with the pinch field. Pinch decay was observed by the author in a rail gun when one of the electrodes was cut out so that on a certain section of the rail its transverse dimension was reduced to the diameter of the pinch, which was much less than the distance between electrodes (this setup was described in [11]).

The instability of a plane current sheet in a plasma has been demonstrated on several occasions [13, 14]. Under experimental conditions the "pinches" move through the plasma. Therefore we will consider the case when a considerable part of the discharge current flows through a series of narrow channels (assuming that for this case the conductivity of the pinches is infinite).

We assume that a plane surface layer has broken down into a series of identical current channels (pinches) over which flows a compressible gas of constant conductivity σ_0. The figure shows the magnetic lines of force around and behind the pinches. The pinches are distributed in the plane zOy; the direction of the current in them is parallel to the z axis. The plasma occupies the half-space $x < l$ and moves along the x axis. The total discharge current is assumed constant. In the plasma behind the pinches there is induced an electromotive force whose direction coincides with the direction of the current in the pinches; therefore there is a redistribution of the current between the pinches and the plasma. Making certain assumptions with respect to the plasma motion, we will find the conditions under which a considerable part of the total discharge current can flow through the pinches.

The boundary conditions are

$$\mathbf{E} = \mathbf{H} = 0, \quad \mathbf{V} = V_0\mathbf{i}$$

$$\rho = \rho_0, \quad p = p_0 \quad \text{when } x = -\infty .$$

At the surface of the plasma

$$\mathbf{H} = H_0\mathbf{j} \qquad (H_0 = \mathrm{const})$$

$$p = \rho = 0 \quad \text{when } x = l\ (l \gg d) .$$

The magnetic pressure is much greater than the gas-kinetic pressure of the unperturbed plasma $H_0^2/8\pi \gg p_0$.

We assume that at a certain distance from the pinches $x > l_1$ ($d < l_1 \ll l$), as a result of the viscosity and deceleration of the plasma by the magnetic field its motion is one-dimensional.

We will estimate the plasma velocity behind the pinches $l_1 \leq x \leq l$, assuming that it does not depend on x and that the current density is constant:

$$H = H_1 + (H_0 - H_1)(x - l_1) / (l - l_1)$$

$$(H_1 = H(l_1) \geqslant {}^1/_2 H_0) .$$

From the equation of the magnetic field

$$\partial H / dt + \partial (VH) / \partial x = c^2 (4\pi\sigma_0)^{-1} \partial^2 H / \partial x^2,$$

it follows that

$$\partial H / \partial t = - V_1 \partial H / \partial x .$$

Then, assuming that near the pinches $\partial H/\partial t$ is of the same order, we integrate the expression for the current density

$$j = c(4\pi)^{-1} \partial H / \partial x = \sigma_0 (E_z + c^{-1} VH)$$

$$\left(E_z = c^{-1} \int_{-\infty}^{x} \partial H_y / \partial t dx\right)$$

with respect to x from l_1 to l.

This gives $H_0 - H_1 = 4\pi c^{-2} \sigma_0 V_1 H_1 l$; consequently, $4\pi c^{-2} \sigma_0 V_1 l < 1$. Equating this with the condition of surface layer formation in the plasma $4\pi c^{-2} \sigma_0 R V_0 \gg 1$, where R is the characteristic dimension of the plasma and R/V_0 the characteristic time of the current sheet collapse process, and considering that $l/V_1 \approx R/V_0$, we obtain $l \ll R$ and $V_1 \ll V_0$, the condition of total deceleration of the plasma flowing over the pinches. Taking into account the increase in plasma conductivity as a result of heating during deceleration only strengthens the latter inequality.

We also assume that the characteristic time Δt of variation of the parameters of the plasma and the field near the pinches $x < l_1$, where deceleration mainly takes place, is of the same order as or greater than the time of the entire process $\Delta t \gtrsim R/V_0$. We may then assume that the deceleration of the plasma is stationary, since the deceleration time $\tau \approx l_1/V_0$ is much less than the time Δt: $\tau \approx l_1/V_0 \ll R/V_0 \lesssim \Delta t$.

The plasma velocity V_0 is assumed to be such that the force accelerating the pinches

$$F = \frac{1}{2c^2} (In)^2 \frac{dL}{dx} \geqslant \left(\frac{H_0}{2}\right)^2 \frac{nd}{8\pi}$$

(where I is the pinch current, n is the number of pinches, and dL/dx is the inductance of unit length of the electrodes) is balanced by the decelerating force associated with the plasma flow. The radius of the pinches is assumed to be sufficiently small as compared with the distance between them ($r \ll d$), for it to be possible to neglect viscous deceleration of the plasma at the surface of the pinches as compared with the inductive deceleration of the plasma by the magnetic field. By this we understand the interaction between the external magnetic field and the currents induced in the plasma as it moves in that field. Thus, the magnetic pressure $H_0^2/8\pi$ acts only on the plasma.

If we neglect the heat flow from the pinches to the plasma and radiative heat transfer as compared with the flow of kinetic and internal energy and assume that the ratio of specific heats of the gas is constant γ = const, then for a stationary plasma flow the laws of conservation of energy, momentum, and mass for $x = -\infty$ and $x = l_1$ can be written in the form

$$\frac{V_0}{2} + \frac{\gamma}{\gamma - 1} \frac{p_0}{\rho_0} = \frac{V_1^2}{2} + \frac{\gamma}{\gamma - 1} \frac{p_1}{\rho_1}$$

$$\rho_0 V_0^2 = H_1^2 / 8\pi + p_1 + \rho_1 V_1^2, \qquad \rho_0 V_0 = \rho_1 V_1 . \qquad (1)$$

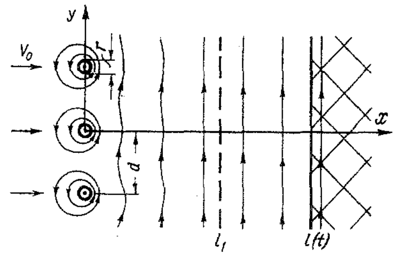
Considering that $V_1 \ll V_0$ and, consequently, $dV_1/dt \approx V_1 V_0/R \ll V_0^2/R$, from the equation of motion for the plasma behind the pinches $x \geq l_1$ we obtain

$$\rho_0 R dV_1 / dt = p_1 - (H_0^2 - H_1^2) / 8\pi \ll \rho_0 V_0^2 .$$

From Eqs. (1) and this inequality it follows that

$$\rho_0 V_0^2 = \frac{H_0^2}{8\pi}, \qquad \frac{\gamma - 1}{2\gamma} < \frac{V_1}{V_0} \ll 1 , \qquad (2)$$

i. e., the ratio of specific heats of the gas must be quite small $\gamma \approx 1$.



The pinches create inhomogeneities in the plasma flow with characteristic dimension along the y axis of the order of the distance between pinches d; therefore currents in the plasma located behind the pinches at a distance greater than d cannot create a magnetic field in the region of the pinches comparable with H_0 and exert an appreciable decelerating influence on them, i. e., the force accelerating the pinches, which is assumed greater than $(H_0/2)^2/8\pi$, is balanced by the reaction of the plasma at a distance of the order of or less than d from the pinches and equal to

$$\iint \frac{1}{c} \mathbf{j} \times \mathbf{H} dx dy > \left(\frac{H_0}{2}\right)^2 \frac{d}{8\pi} \qquad \left(\mathbf{j} = \frac{\sigma_0}{c} \mathbf{V} \times \mathbf{H}\right) .$$

where the integral is taken over the area $-kg/2 \leq x \leq kd/2$, $-d/2 \leq y \leq d/2$ (k is a coefficient of the order of unity). An estimate of the integral from the maximum gives

$$\frac{\sigma_0}{c^2} \iint [\mathbf{V} \times \mathbf{H}] \times \mathbf{H} dx dy < k \frac{\sigma_0}{c^2} \left(\frac{H_0}{2}\right)^2 V_0 d ,$$

and, consequently, $k 8\pi c^{-2} \sigma_0 V_0 d > 1$.

After eliminating V_0 from this inequality, using (2), we obtain

$$\rho_0^{1/2} < k (8\pi)^{1/2} c^{-2} \sigma_0 H_0 d . \qquad (3)$$

Thus, deceleration of the plasma by the magnetic field of the pinches takes place only at a sufficiently small gas density. From this there also follows the existence of a minimum distance between pinches, and, consequently, a maximum number of pinches, which is proportional to H_0 or the discharge current.

The previously obtained condition $\gamma \approx 1$ is, in fact, the lower bound of the region of existence of the pinches with respect to gas density, since to satisfy $\gamma \approx 1$ the characteristic time of the excitation and ionization processes must be small as compared with the plasma deceleration time $\tau_i \ll d/V_0 = (8\pi)^{1/2} dH_0^{-1}\rho_0^{1/2}$. The characteristic ionization time is inversely proportional to the atom concentration, the ionization cross section, and the electron velocity

$$\frac{1}{\tau_i} = n_a \langle \sigma_i v_e \rangle, \qquad \langle \sigma_i v_e \rangle = \int_{v_e^*}^{\infty} \sigma_i(v_e)\, v_e f(v_e)\, dv_e \ .$$

Here, $f(v_e)$ is the electron velocity distribution function; $(1/2)m_e v_e^{*2} = E_i$ is the ionization energy. There is an analogous expression for excitation processes. When only elastic collisions are taken into account, the electron temperature of the decelerated plasma is proportional to the square of the velocity V_0 and the atomic weight of the ions M, but the product $\langle \sigma_i v_e \rangle$ has a maximum with increase in electron temperature $\langle \sigma_i v_e \rangle_*$; therefore for the density we have the lower bound

$$\rho_0^{3/2} \gg \frac{MH_0}{\langle \sigma_i v_e \rangle_* d} \ . \qquad (4)$$

When the kinetic energy of the atoms $(1/2)MV_0^2$ is less than the energy of the electrons at which $\sigma_i(v_e)$ has a maximum, for estimating purposes it is better to take an electron temperature in expression (4) equal to the temperature of the decelerated plasma with account for elastic collisions only.

This value of the minimum density is much too low, since heating of the plasma electrons due to deceleration may increase the plasma conductivity to values comparable with the conductivity of the pinches, and, to judge from the measured value of the temperature in the pinches [11], this occurs long before the gas-kinetic pressure in the decelerated plasma becomes equal to the magnetic pressure $H_0^2/8\pi$. Therefore we will obtain the minimum density more accurately if in the right side of inequality (4) we substitute the temperature of the electrons in the pinches. In order for a large part of the main current to flow through the pinches, their resistance must be of the same order as or less than the resistance of the decelerated plasma $\sigma\pi r^2 \gtrsim \sigma_1 R d V_1/V_0$, where σ and r are the conductivity and radius of the pinches, σ_1 the conductivity of the decelerated plasma. For sufficiently complete deceleration $V_1/V_0 \ll 1$ this condition will be satisfied when the conductivity of the pinch plasma is not much greater than the conductivity of the decelerated plasma. Ahead of the pinches $x \ll -d$ the current is equal to zero, since $E = H = 0$.

The relation obtained $V_1 \ll V_0$ also relates to the case $\omega_e \tau_e \gg 1$ (ω_e is the Larmor frequency of the electrons and τ_e is the time between collisons), since in the plasma behind the pinches there is no Hall current ($j_x = 0$). But, generally speaking, near the pinches the Hall current is not zero; therefore conductivity anisotropy can only strengthen inequality (3).

For the deceleration of a weakly ionized plasma, in which the atomic mean free path is large as compared with d, the kinetic energy of the atoms must be sufficient for their ionization $(1/2)MV_0^2 > E_i$ or the magnetic field must be sufficiently strong $H_0^2 > 16\pi\rho_0 E_i M^{-1}$.

Experimental investigation of the Z pinch [7] has revealed good agreement with the value of the current sheet velocity calculated from the "snowplow" model, irrespective of whether the current sheet is homogeneous or consists of separate channels. The drag of the gas flow can be neglected, since $r \ll d$. The viscous deceleration is also small ($N_{Re} \gg 1$), whereas numerical estimates employing discharge parameters taken from [6–9] show that the region of existence of the pinches satisfies inequalities (3) and (4) obtained from the condition of total deceleration of the plasma by the magnetic field of the pinches. According to [9], this region is bounded by the pressures: $0.5 > p_0 > 0.1$ mm Hg in oxygen and nitrogen, $10 > p_0 > 0.5$ mm Hg in hydrogen, $0.1 > p_0 > 0.025$ mm Hg in argon, and $1 > p_0 > 0.5$ mm Hg in helium. In accordance with (3), the upper limit is approximately inversely proportional to the molecular weight of the gas. The lower pressure is less for gases with a large molecular weight and large ionization cross section (argon) and correspondingly greater for light gases with small ionization cross sections (hydrogen and helium). Numerical estimates for argon are presented below.

The basic discharge parameters of [6–9] are: tube diameter 2R = = 15 cm, tube length 62 cm, the maximum current in the first half-period was about $2 \cdot 10^5$ A and was reached in about $3 \cdot 10^{-6}$ sec, the number of pinches $n \approx 10$ ([7], Fig. 1). We will take all the discharge parameters at the moment when the current sheet is at a sufficient distance from the walls and has a radius 0.7R ([7], Fig. 2) (closer to the axis the accuracy of the pinch velocity and position measurements decreases). The distance between pinches, the total discharge current, and the magnetic field have the following values:

$$d = 2\pi\ 0.7\ Rn^{-1} = 3.3\,\mathrm{cm}, \quad In = 1.3 \cdot 10^5\,\mathrm{A}$$
$$H_0 = 2c^{-1}\ In\ (0.7\ R)^{-1} = 5.24 \cdot 10^3\,\mathrm{Oe}\ .$$

The plasma conductivity was not measured; therefore we will estimate it from the condition of surface layer formation $\sigma_0 \gg c^2(4\pi)^{-1}T \times$ $\times R^{-2} = 5 \cdot 10^{12}$ CGSE units, where $T \approx 4 \cdot 10^{-6}$ sec is the current sheet collapse time. We assume that $\sigma_0 = 5c^2(4\pi)^{-1}TR^{-2} = 2.5 \cdot 10^{13}$ CGSE units, which corresponds to the conductivity at an electron temperature of about 2.5 eV. Substituting for σ_0, d, and H_0 in (3), we obtain $\rho_0 <$ $< 5.5 \cdot 10^{-6}\ \mathrm{g \cdot cm^{-3}}$. The density of argon at an initial pressure 0.1 mm Hg is $\rho_0 = 2.35 \cdot 10^{-7}\ \mathrm{g \cdot cm^{-3}}$.

At the minimum argon pressure $p_0 = 0.025$ mm Hg the characteristic ionization time τ_i must be small as compared with the plasma deceleration time $\tau_i \ll d/V_0 \approx 1.4 \cdot 10^{-6}$ sec. In these experiments the pinch plasma temperature was not measured; therefore we will estimate τ_i at an electron temperature equal to the plasma stagnation temperature. The pinch velocity $V_0 = 2.3 \cdot 10^6$ cm/sec, observed at this pressure, corresponds to a kinetic energy of the argon atoms equal to about 110 eV; therefore during the deceleration time the argon is triply ionized, and the deceleration temperature is equal to about $4 \cdot 10^{4\circ}$ K.

When the electron temperature is sufficiently small $kT_e \ll E_i$, the ionization cross section can be used in the form $\sigma_i(v_e) \approx C(1/2 m_e v_e^2 - E_i)$. Integration of the expression for τ_i gives

$$\frac{1}{\tau_i} = cp_0 \left(\frac{8kT_e}{\pi m_e}\right)^{1/2} (E_i - 2kT_e) \exp\left(-\frac{E_i}{kT_e}\right).$$

Substituting the value of the ionization energy of argon E_i = = 15.7 eV, the inelastic collision cross section constant $C = 0.7\ \mathrm{cm^{-1}} \cdot$ $\cdot\ \mathrm{V \cdot mm\ Hg^{-1}}$ [15], pressure $p_0 = 0.025$ mm Hg, and $T_e = 5 \cdot 10^{4\circ}$ K, we obtain $\tau_i = 2.2 \cdot 10^{-6}$ sec. If we assume that double and triple ionization of argon takes place in steps and that the slowest process is transition from the ground state to the first excited state, the ionization time for Ar^+ and Ar^{2+} is of the same order, since the first excitation potentials of Ar^+ and Ar^{2+} are small (about 15 eV), and the constants of the excitation cross section are usually of the same order. Since τ_i decreases rapidly as T_e increases, the electron temperature cannot greatly exceed the argon stagnation temperature with account for ionization; therefore γ may be assumed small.

When conditions (3) and (4) are not satisfied, the time of existence of the pinches into which the plane current sheet is divided is small as compared with the time of the process R/V_0; consequently, in these pressure ranges the instability of the current sheet is chiefly manifested as an erosion in the direction of motion.

The pinches break down as a result of the decrease in the current in them created by the plasma through which the pinches move when it is incompletely decelerated. This applies mainly to θ pinches, where the currents are closed in the plasma, and also to Z pinches and linear plasma accelerators at a sufficiently large initial pressure and capacitor voltage. However, it is known [16, 17] that at low pressures, small interelectrode spacings, and short voltage pulses the voltage associated with an arc discharged through a gas is large in the presence of considerable ionization. If we assume that the electric field of the space charges near the electrodes is compensated by the induced emf $c^{-1}[VH]$, so that the total current to the electrodes through the plasma flowing over the pinches becomes negligibly small (the plasma is isolated from the electrodes and the pinches), and consider only closed currents in it, then, generally speaking, complete raking up of the plasma by the pinches is not absolutely necessary [11]. In linear accelerators, where the voltage between the electrodes is not sufficient for an arc discharge through the plasma surrounding the pinches, the closed currents flow mainly near the pinches; therefore the pinch structure of the current sheet may be the reason for a broad spectrum of velocities in the plasmoid, as in [11].

According to Earnshaw's theorem [18] the current channels cannot be in stable equilibrium under the action of magnetic forces alone. But experiment shows that the network of current filaments is quite stable throughout the entire process of collapse of the current sheet [5, 6]. This stability may be associated with the induced currents in the plasma flowing over the pinches. As the distance between two pinches increases, as compared with the mean, the fraction of plasma decelerated behind them increases, i. e., between these pinches there appears a supplementary current whose direction coincides with the pinch current, which means an additional force attracting the pinches.

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